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OPTIMAL BACKUP STRATEGY FOR MAKING FILES BY COMPUTER

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Abstract When we make document files by a computer or a wordprocessor, we sometimes meet with the accident that all the files having been stored in the floppy disk are lost and cannot be reconstructed. In order to avoid this kind of accident, we sometimes make a backup copy of the floppy disk which is useful when the original floppy disk is broken. We make a stochastic model which is useful to make a decision about whether or not to make a backup disk when there are k files in the floppy disk whose backup copy does not exist, we should make remaining n new files, and the probability that the accident occurs is p. The problem is formulated by dynamic programming and several properties of the optimal strategy are obtained. The case that the true value of p is unknown is also discussed and several properties have been obtained.

1. Introduction

Wordprocessors and personal computers are very useful and play important roles in the offices and in our daily lives. We create many document files such as letters and articles by using them. After making files by computers and wordprocessors, we sometimes store them in the floppy disks or other media and usually use them again in the near future. Files on the floppy disks are sometimes broken by several accidents and in this case we have to make the same files from the beginning. In order to avoid this kind of additional work, we usually make a backup copy of the original floppy disk to keep it in other place and use it if the files in the original floppy disk are broken. Since to make a backup copy of the floppy disk whenever we make a new file on it is time-consuming, we make a backup copy of the floppy disk only after several new files, whose copies do not exist anywhere, have been made.

The idea to make the backup copy of files is a famous and important idea which we have been using in the works of computers. The time point to save the current program executed up to the point is called a checkpoint and when to place the checkpoint is the important problem and discussed in several papers, for example, Young[19], Chandy and Ramamoorthy [5], Chandy, Browne, Dissly and Uhrig [4], Gelenbe [9], Kaio and Osaki [12], Toueg and Babaoğlu [18], Sumita, Kaio and Goes [17], Goes and Sumita [11], Dohi, Aoki, Kaio and Osaki [7], and so on. A kind of backup policy is recently discussed in Sandoh and Kawai [15] and they analyzed the model as the renewal reward process and considered N-job backup policy in which a backup copy is made only after completeing every N jobs and how to determine the optimal value of N. Backup policies for hard computer disks are also discussed in Sandoh, Kaio and Kawaii [14] and Sandoh, Kawaii and Ibaraki [16].

In this paper, we consider the problem that there are k files in the floppy disk whose backup copies do not exist and we have to make additional n new files remaining. The probability that both the k files which have been kept in a floppy disk and the additional file which is currently being constructed will be lost by an accident is given by p. We consider the cost of making a copy of a floppy disk and that of reproducing each file which is lost by an accident. We also consider the reward for each file whose copy exists in the other floppy disk. The problem is formulated by the stochastic dynamic programming (about which, see, for example, Ross [13]).

When the value of p is unknown, we have to acquire the information about its value and the problem has some similarity with the two-armed bandit problem with one arm known. There are many papers about Bernoulli two-armed bandit problem, for example, Bradt, Johnson and Karlin [3], Feldman [8], Berry [1], and so on. Also, many bibliographies can be found in Berry and Fristedt [2] and Gittins [10].

We consider the problem as a stochastic sequential decision problem to determine whether to make a backup disk or not, and then formulate it by the principle of optimality of dynamic programming. Several properties are derived in Section 2. The extended model with an unknown parameter is discussed in Section 3. Some relations between the model of this paper and the two-armed bandit problem are discussed in Section 4.

2. Model

Suppose that we have to make n files by a computer or a wordprocessor and we have just completed making the first one of them. Now, we are ready to store it in the floppy disk in which there are already k files whose backup copies do not exist and there is a sufficiently large memory space remaining. We will denote this state by (n, k). After saving it in the floppy disk, we have to select one of two actions, a_0 or a_1, where a_0 is to make the backup copy of the floppy disk and a_1 is not to make it.

Consider the case that action a_0 is selected in state (n, k). Then, since making a backup copy needs an additional work, we evaluate the cost of making a backup copy as C. Once the backup floppy disk is made, the $k+1$ files can be restored and kept to be safe. In this case, we evaluate the reward of safety of each file as R. Therefore, the total reward of making a backup copy is $-C+(k+1)R$ and the new state at the next stage becomes $(n-1, 0)$.

Also, consider the other case that action a_1 is selected in state (n, k). Then, since there are $k+1$ files whose backup copies do not exist, these files will be lost with positive probability p and will not be lost with probability $1-p$, where p is the probability that the floppy disk will be lost or its contents will be broken or lost between the time to make a decision of this period and that of the next period. If $k+1$ files are lost by an accident, the state at the next stage will be $(n-1, 0)$ and we have to make these $k+1$ files again from the outset. In this case, the additional work is evaluated by the cost B incurred for each file, that is, the total cost is $(k+1)B$ if $k+1$ files are lost. Also, if the $k+1$ files are not lost, the state at the next stage will be $(n-1, k+1)$ and no reward is obtained in this case.

Let $f_n(k)$ be the maximum expected total reward over n stages when the current state is (n, k). Also, let $f_n^i(k)$ $(i=0,1)$ be the maximum expected total reward over n stages if action a_i is selected in state (n, k) and the optimal strategy is followed thereafter. Then, the following recursive equations are obtained:

$$f_n(k) = \max\left\{f_n^0(k), f_n^1(k)\right\} \tag{2.1}$$

for $n = 1, 2, 3, \cdots$ and $k = 0, 1, 2, \cdots$ and

$$f_0(k) = kR \tag{2.2}$$

for $k = 0, 1, 2, \cdots$, where

$$f_n^0(k) = -C + (k+1)R + f_{n-1}(0) \tag{2.3}$$

and

$$f_n^1(k) = p\{-(k+1)B + f_{n-1}(0)\} + (1-p)f_{n-1}(k+1). \tag{2.4}$$

To select $a_i (i = 0, 1)$ is optimal in state (n, k) if $f_n^i(k) \geq f_n^{1-i}(k)$. Although both a_0 and a_1 are optimal in state (n, k) if $f_n^0(k) = f_n^1(k)$, we say a_0 is optimal in this state as the matter of convenience.

Lemma 1 *For* $n = 0, 1, 2, \cdots$ *and* $k = 1, 2, 3, \cdots$,

$$-B \leq f_n(k) - f_n(k-1) \leq R \tag{2.5}$$

and therefore

$$-kB \leq f_n(k) - f_n(0) \leq kR. \tag{2.6}$$

Proof. The assertion is trivial for $n = 0$ and $k = 1, 2, 3, \cdots$. Assume that

$$-B \leq f_{n-1}(k) - f_{n-1}(k-1) \leq R$$

for $n \geq 1$ and $k = 1, 2, 3, \cdots$. Then

$$f_n^0(k) - f_n^0(k-1) = R$$

and

$$f_n^1(k) - f_n^1(k-1) = -pB + (1-p)\{f_{n-1}(k+1) - f_{n-1}(k)\}.$$

From the inductive hypothesis that

$$-B \leq f_{n-1}(k+1) - f_{n-1}(k) \leq R,$$

we derive

$$-B \leq f_n^1(k) - f_n^1(k-1) \leq -pB + (1-p)R.$$

Therefore,

$$\begin{aligned} f_n(k) &= \max\{f_n^0(k), f_n^1(k)\} \\ &\leq \max\{f_n^0(k-1) + R, f_n^1(k-1) - pB + (1-p)R\} \\ &\leq \max\{f_n^0(k-1) + R, f_n^1(k-1) + R\} \\ &= R + f_n(k-1). \end{aligned}$$

Also,

$$\begin{aligned} f_n(k) &\geq \max\{f_n^0(k-1) + R, f_n^1(k-1) - B\} \\ &\geq \max\{f_n^0(k-1) - B, f_n^1(k-1) - B\} \\ &= -B + f_n(k-1). \end{aligned}$$

This completes the proof of (2.5). (2.6) is the immediate consequence of (2.5).□

Now let

$$d_n(k) = f_n^0(k) - f_n^1(k)$$

for $n = 1, 2, 3, \cdots$ and $k = 0, 1, 2, \cdots$. Then, a_0 is optimal if and only if $d_n(k) \geq 0$. By using (2.3) and (2.4),

$$d_n(k) = -C + (k+1)(R + pB) + (1-p)\{f_{n-1}(0) - f_{n-1}(k+1)\} \tag{2.7}$$

which is rewritten as follows:

$$\begin{aligned} (B+R)^{-1}d_n(k) &= -C(B+R)^{-1} + p(k+1) \\ &+ (1-p)(B+R)^{-1}\{f_{n-1}(0) - f_{n-1}(k+1) + (k+1)R\}, \end{aligned} \tag{2.8}$$

The function $d_n(k)$ has the following property.

Lemma 2 *For $n \geq 1$ and $k \geq 0$,*
(i) $(B+R)^{-1}\{f_{n-1}(0) - f_{n-1}(k+1) + (k+1)R\}$ *is a function of p and $C(B+R)^{-1}$.*
(ii) $(B+R)^{-1}d_n(k)$ *is a function of p and $C(B+R)^{-1}$.*

Proof. Since

$$(B+R)^{-1}\{f_0(0) - f_0(k+1) + (k+1)R\} = 0,$$

(i) is true for $n = 1$. For $n \geq 2$, assume that $(B+R)^{-1}\{f_{n-2}(0) - f_{n-2}(k+1) + (k+1)R\}$ is a function of p and $C(B+R)^{-1}$. Then,

$$(B+R)^{-1}\{f_{n-1}(0) - f_{n-1}(k+1) + (k+1)R\}$$

$$= \max\left[-C(B+R)^{-1}, -p - (1-p)(B+R)^{-1}\{f_{n-2}(0) - f_{n-2}(1) + R\}\right]$$

$$- \max\left[-C(B+R)^{-1}, -p(k+2) - (1-p)(B+R)^{-1}\{f_{n-2}(0) - f_{n-2}(k+2) + (k+2)R\}\right].$$

From the inductive hypothesis, the right hand side of this equation is also a function of p and $C(B+R)^{-1}$ and therefore (i) is true for $n \geq 1$. From (2.8), (ii) is the immediate consequence of (i).□

Lemma 3 *For $n = 1, 2, 3, \cdots$ and $k = 1, 2, 3, \cdots$,*

$$p(B+R) \leq d_n(k) - d_n(k-1) \leq B+R. \tag{2.9}$$

Proof. From (2.7),

$$d_n(k) - d_n(k-1) = R + pB - (1-p)\{f_{n-1}(k+1) - f_{n-1}(k)\}.$$

Using Lemma 1,

$$R + pB - (1-p)R \leq d_n(k) - d_n(k-1) \leq R + pB + (1-p)B,$$

that is,

$$p(B+R) \leq d_n(k) - d_n(k-1) \leq B+R$$

which completes the proof.□

It is derived from this lemma that

$$d_n(k-1) < d_n(k)$$

for $n = 1, 2, 3, \cdots$ and $k = 1, 2, 3, \cdots$. Since $(B+R)^{-1}d_n(k)$ is a function of p and $C(B+R)^{-1}$, let $k_n^*(p, C(B+R)^{-1})$ be the smallest k that satisfies $d_n(k) \geq 0$. Then, to make a backup file is optimal in state (n, k) if and only if $k \geq k_n^*(p, C(B+R)^{-1})$.

Theorem 1 *For $n \geq 1$ and $k \geq 1$,*
(i) *if it is optimal to make a backup file in state $(n, k-1)$, then it is also optimal in state (n, k) and*
(ii) *if it is not optimal to make a backup file in state (n, k), then it is also not optimal in state $(n, k-1)$.*

Proof. (i) is immediate from the first inequality of (2.9) which means that $d_n(k) \geq 0$ if $d_n(k-1) \geq 0$. (ii) is the contraposition of (i).□

Lemma 4 *For $n = 1, 2, 3, \cdots$ and $k = 1, 2, 3, \cdots$,*

$$-C + p(k+1)(B+R) \leq d_n(k) \leq -C + (k+1)(B+R). \tag{2.10}$$

Proof. (2.10) is immediate from (2.7) and

$$-(k+1)R \leq f_{n-1}(0) - f_{n-1}(k+1) \leq (k+1)B,$$

which is derived from (2.6) of Lemma 1 .□

From this lemma, we derive other properties of the optimal strategy which are given in the following theorem.

Theorem 2 *(i) If $k \geq C\{p(B+R)\}^{-1} - 1$, then to make a backup file is optimal. (ii) If $k < C(B+R)^{-1} - 1$, then to make a backup file is not optimal.*

From this theorem, we derive that

$$C(B+R)^{-1} - 1 \leq k_n^* \left(p, C(B+R)^{-1}\right) \leq C\{p(B+R)\}^{-1} - 1.$$

The values of $k_n^* (p, C(B+R)^{-1})$ for $C(B+R)^{-1} = 1/5, 1/4, 1/3, 1/2, 1, 2, 3, 4, 5$ are calculated by using a computer and tabulated in Table 1.

3. Case that the value of p is unknown

In this section, we consider the case that the true value of p is unknown and is given the beta prior distribution with parameters (s, t) whose density function is given by

$$g(p|s,t) = \begin{cases} \dfrac{\Gamma(s+t)}{\Gamma(s)\Gamma(t)} p^{s-1}(1-p)^{t-1}, & if\ 0<p<1, \\ 0, & otherwise. \end{cases}$$

Although the true value of p is generally small, it can not be neglected and therefore we assume that the value of t is sufficiently larger than that of s at the outset. Since beta distribution is a conjugate family for a sample from a Bernoulli distribution, the posterior distribution of p becomes beta distribution with parameters $(s+1, t)$ or $(s, t+1)$ if an accident ocuurs or not, respectively (See, for example, Chapter 9 of DeGroot [6]). Therefore, the state is denoted by $(n, k; s, t)$.

If a_0 is selected in state $(n, k; s, t)$, the state will make a transition to state $(n-1, 0; s, t)$ with probability 1. In this case, the prior information about the true value of p remains unchanged and all the files constructed up to this point have backup copies.

If a_1 is selected in state $(n, k; s, t)$ and an accident occurs, the state will make a transition to state $(n-1, 0; s+1, t)$ and the expected transition probability from $(n, k; s, t)$ to $(n-1, 0; s+1, t)$ is $s/(s+t)$. Also, if a_1 is selected in state $(n, k; s, t)$ and no accident occurs, the next state will be $(n-1, k+1; s, t+1)$ and the expected transition probability in this case is $t/(s+t)$.

Let $f_n(k; s, t)$ be the maximum expected total reward over n stages when the current state is $(n, k; s, t)$. Also, let $f_n^i(k; s, t)$ $(i = 0, 1)$ be the maximum expected total reward over n stages if action a_i is selected in state $(n, k; s, t)$ and the optimal strategy is followed thereafter. Then, the following recursive equations are obtained:

$$f_n(k; s, t) = \max\left\{f_n^0(k; s, t), f_n^1(k; s, t)\right\} \tag{3.1}$$

Table 1. Values of $k_n^*(p, C(B+R)^{-1})$ for $n = 1, 2, \cdots, 15$, $p = 1/10, 1/20, 1/30, 1/50, 1/100$, and $C(B+R)^{-1} = 1/5, 1/4, 1/3, 1/2, 1, 2, 3, 4, 5$.

p	$\frac{C}{B+R}$ \ n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1/10	1/5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1/4	2	1	1	1	1	1	1	1	1	1	1	1	1	1	1
	1/3	3	1	2	2	2	2	2	2	2	2	2	2	2	2	2
	1/2	4	2	2	2	3	2	2	2	2	2	2	2	2	2	2
	1	9	5	3	3	4	4	4	3	4	4	4	4	4	4	4
	2	19	10	7	5	5	5	6	6	7	7	5	6	6	6	6
	3	29	15	11	8	7	6	7	7	8	8	8	9	8	7	7
	4	39	21	14	11	9	8	8	8	9	9	9	10	10	10	10
	5	49	26	18	14	12	10	9	9	10	10	10	11	11	11	11
1/20	1/5	3	2	2	2	2	2	2	2	2	2	2	2	2	2	2
	1/4	4	2	2	2	3	2	2	2	2	2	2	2	2	2	2
	1/3	6	3	2	3	3	3	3	3	3	3	3	3	3	3	3
	1/2	9	5	3	3	4	4	4	3	4	4	4	3	4	4	4
	1	19	10	7	5	4	5	6	6	7	6	5	5	6	6	6
	2	39	20	14	10	8	7	7	7	8	9	9	10	10	9	8
	3	59	30	21	16	13	11	9	8	9	10	10	11	11	12	12
	4	79	41	28	21	17	15	13	11	10	11	11	12	12	13	13
	5	99	51	35	26	22	18	16	14	13	12	12	13	13	14	14
1/30	1/5	5	3	2	3	3	2	3	3	3	3	3	3	3	3	3
	1/4	7	3	2	3	3	3	3	3	3	3	3	3	3	3	3
	1/3	9	5	3	3	4	4	4	3	4	4	4	3	4	4	4
	1/2	14	7	5	3	4	5	5	6	4	4	5	5	5	5	4
	1	29	15	10	7	6	5	6	7	7	8	9	7	6	6	7
	2	59	30	20	15	12	10	9	8	8	9	10	11	11	12	12
	3	89	45	31	23	19	16	14	12	11	10	11	12	12	13	13
	4	119	61	41	31	25	21	18	16	15	13	12	13	13	14	14
	5	149	76	51	39	32	27	23	21	19	17	16	14	14	15	15
1/50	1/5	9	5	3	3	4	4	4	3	4	4	4	3	4	4	4
	1/4	12	6	4	3	4	4	5	4	4	4	4	4	4	4	4
	1/3	16	8	5	4	4	5	5	6	5	4	5	5	5	5	5
	1/2	24	12	8	6	5	5	6	6	7	8	6	5	6	6	6
	1	49	25	17	12	10	8	7	7	8	9	9	10	11	12	11
	2	99	50	34	25	20	17	15	13	12	10	10	11	12	13	14
	3	149	75	51	38	31	26	22	20	18	16	15	13	13	14	15
	4	199	101	68	51	41	35	30	26	24	21	20	18	17	16	16
	5	249	126	85	64	52	43	37	33	30	27	25	23	21	20	19
1/100	1/5	19	10	6	5	4	5	5	6	7	5	5	5	5	5	6
	1/4	24	12	8	6	5	5	6	6	7	8	6	5	6	6	6
	1/3	33	16	11	8	6	5	6	7	7	8	9	9	7	6	7
	1/2	49	25	16	12	10	8	7	7	8	9	9	10	11	12	10
	1	99	50	33	25	20	17	14	12	11	10	10	11	12	13	13
	2	199	100	67	50	40	34	29	25	23	20	19	17	16	15	14
	3	299	150	101	76	61	51	44	38	34	31	28	26	24	22	21
	4	399	201	134	101	81	68	58	51	46	41	38	35	32	30	28
	5	499	251	168	126	102	85	73	64	57	52	47	44	40	38	35

for $n=1,2,3,\cdots$ and $k=0,1,2,\cdots$ and

$$f_0(k;s,t)=kR \tag{3.2}$$

for $k=0,1,2,\cdots$, where

$$f_n^0(k;s,t)=-C+(k+1)R+f_{n-1}(0;s,t) \tag{3.3}$$

and

$$f_n^1(k;s,t)=\frac{s}{s+t}\{-(k+1)B+f_{n-1}(0;s+1,t)\}+\frac{t}{s+t}f_{n-1}(k+1;s,t+1). \tag{3.4}$$

To select $a_i(i=0,1)$ is optimal in state $(n,k;s,t)$ if $f_n^i(k;s,t)\geq f_n^{1-i}(k;s,t)$. Although both a_0 and a_1 are optimal in state $(n,k;s,t)$ if $f_n^0(k;s,t)=f_n^1(k;s,t)$, we say a_0 is optimal in this case as the matter of convenience. Now let

$$d_n(k;s,t)=f_n^0(k;s,t)-f_n^1(k;s,t)$$

for $n=1,2,3,\cdots$ and $k=0,1,2,\cdots$. Then,

$$d_n(k;s,t)=-C+(k+1)(R+\frac{s}{s+t}B)+f_{n-1}(0;s,t)$$
$$-\frac{s}{s+t}f_{n-1}(0;s+1,t)-\frac{t}{s+t}f_{n-1}(k+1;s,t+1). \tag{3.5}$$

This is rewritten as follows:

$$(B+R)^{-1}d_n(k;s,t)=-C(B+R)^{-1}+\frac{s}{s+t}(k+1)+W_n(k+1;s,t;B,R,C), \tag{3.6}$$

where

$$W_n(l;s,t;B,R,C)=(B+R)^{-1}\left\{f_{n-1}(0;s,t)-\frac{s}{s+t}f_{n-1}(0;s+1,t)\right.$$
$$\left.-\frac{t}{s+t}f_{n-1}(l;s,t+1)+\frac{t}{s+t}lR\right\}.$$

for $l=0,1,2,\cdots$.

Lemma 5 *For $n\geq 1$ and $l\geq 0$,*
(i) *$W_n(l;s,t;B,R,C)$ is rewritten as a function of s, t and $(B+R)^{-1}C$.*
(ii) *$(B+R)^{-1}d_n(l;s,t)$ is also a function of s, t and $(B+R)^{-1}C$.*

Proof. Since

$$W_1(l;s,t;B,R,C)=0,$$

(i) is true for $n=1$. For $n\geq 2$, assume that $W_{n-1}(l;s,t;B,R,C)$ is rewritten as a function of s, t and $(B+R)^{-1}C$. Then, since

$$(B+R)^{-1}f_{n-1}(0;s,t)=(B+R)^{-1}R+(B+R)^{-1}f_{n-2}(0;s,t)$$
$$+\max\left\{-C(B+R)^{-1},-\frac{s}{s+t}-W_{n-1}(1;s,t;B,R,C)\right\},$$
$$(B+R)^{-1}f_{n-1}(0;s+1,t)=(B+R)^{-1}R+(B+R)^{-1}f_{n-2}(0;s+1,t)$$
$$+\max\left\{-C(B+R)^{-1},-\frac{s+1}{s+t+1}-W_{n-1}(1;s+1,t;B,R,C)\right\},$$

and

$$(B+R)^{-1}f_{n-1}(l;s,t+1) = (B+R)^{-1}(l+1)R + (B+R)^{-1}f_{n-2}(0;s,t+1)$$
$$+\max\left\{-C(B+R)^{-1}, -\frac{s}{s+t+1}(l+1) - W_{n-1}(l+1;s,t+1;B,R,C)\right\},$$

$W_n(l;s,t;B,R,C)$ is rewritten as follows:

$$W_n(l;s,t;B,R,C) = W_{n-1}(0;s,t;B,R,C)$$
$$+\max\left\{-C(B+R)^{-1}, -\frac{s}{s+t} - W_{n-1}(1;s,t;B,R,C)\right\}$$
$$-\frac{s}{s+t}\max\left\{-C(B+R)^{-1}, -\frac{s+1}{s+t+1} - W_{n-1}(1;s+1,t;B,R,C)\right\}$$
$$-\frac{t}{s+t}\max\left\{-C(B+R)^{-1}, -\frac{s}{s+t+1}(l+1) - W_{n-1}(l+1;s,t+1;B,R,C)\right\}$$

From the inductive hypothesis, the right hand side of this equation is a function of s, t and $(B+R)^{-1}C$, and therefore (i) is true for $n \geq 1$. From (3.6), (ii) is the immediate consequence of (i).□

Now, we can derive the following two lemmas whose proof is almost the same as that of Lemmas 1 and 3 respectively except that p is replaced by $s/(s+t)$.

Lemma 6 *For $n = 0,1,2,\cdots$ and $k = 1,2,3,\cdots$,*

$$-B \leq f_n(k;s,t) - f_n(k-1;s,t) \leq R \tag{3.7}$$

and therefore

$$-kB \leq f_n(k;s,t) - f_n(0;s,t) \leq kR. \tag{3.8}$$

Lemma 7 *For $n = 0,1,2,\cdots$ and $k = 1,2,3,\cdots$,*

$$\frac{s}{s+t}(B+R) \leq d_n(k;s,t) - d_n(k-1;s,t) \leq B+R. \tag{3.9}$$

From Lemma 7,

$$d_n(k-1;s,t) < d_n(k;s,t)$$

for $n = 1,2,3,\cdots$, $k = 0,1,2,\cdots$, $s \geq 1$ and $t \geq 1$. Let $k_n^*(s,t;C(B+R)^{-1})$ be the smallest k that satisfies $d_n(k;s,t) \geq 0$. Then, to make a backup file is optimal in state $(n,k;s,t)$ if and only if $k_n^*(s,t;C(B+R)^{-1}) \leq k$. Also, we obtain the following theorem which gives the main results for the case that the value of the parameter is unknown. Its proof is similar to that of Theorem 1.

Theorem 3 *For $n \geq 1, k \geq 1, s \geq 1$ and $t \geq 1$,*
(i) if it is optimal to make a backup file in state $(n,k-1;s,t)$, then it is also optimal in state $(n,k;s,t)$ and
(ii) if it is not optimal to make a backup file in state $(n,k;s,t)$, then it is also not optimal in state $(n,k-1;s,t)$.

The following lemma gives the upper and the lower bounds for the function $d_n(k;s,t)$ whose proof is in the similar manner as that of Lemma 4.

Lemma 8 *For $n = 1, 2, 3, \cdots$ and $k = 1, 2, 3, \cdots$,*

$$-C + \frac{s}{s+t}(k+1)(B+R) \leq d_n(k; s, t) \leq -C + (k+1)(B+R). \tag{3.10}$$

The optimal strategy has the properties given in the following theorem which are the immediate consequence of Lemma 8.

Theorem 4 (i) *If $k \geq C(s+t)\{s(B+R)\}^{-1} - 1$, then to make a backup file is optimal.* (ii) *If $k < C(B+R)^{-1} - 1$, then to make a backup file is not optimal.*

From this theorem,

$$C(B+R)^{-1} - 1 \leq k_n^*(s, t; C(B+R)^{-1}) \leq C\left\{\frac{s}{s+t}(B+R)\right\}^{-1} - 1.$$

The maximum expected reward $f_n(k; s, t)$ for the state $(n, k; s, t)$ has the properties given in the following theorem.

Theorem 5 *For $n = 0, 1, 2, \cdots$ and $k = 1, 2, 3, \cdots$,*

$$f_n(k; s+1, t) \leq f_n(k; s, t) \tag{3.11}$$

and

$$f_n(k; s, t+1) \geq f_n(k; s, t). \tag{3.12}$$

Proof. Both (3.11) and (3.12) are trivial for $n = 0$. For $n = 1$, it is easily found that

$$f_1^0(k; s+1, t) = f_1^0(k; s, t)$$

and

$$f_1^1(k; s+1, t) = f_1^1(k; s, t) - \frac{t}{(s+t)(s+t+1)}(k+1)(B+R),$$

we will derive that

$$\begin{aligned} f_1(k; s+1, t) &= \max\{f_1^0(k; s+1, t), f_1^1(k; s+1, t)\} \\ &\leq \max\{f_1^0(k; s, t), f_1^1(k; s, t)\} \\ &= f_1(k; s, t), \end{aligned}$$

and by the same way,

$$f_1(k; s, t+1) \geq f_1(k; s, t).$$

Therefore, (3.11) and (3.12) are true for $n = 1$. Assume that both

$$f_{n-1}(k; s+1, t) \leq f_{n-1}(k; s, t)$$

and

$$f_{n-1}(k; s, t+1) \geq f_{n-1}(k; s, t)$$

hold for $n \geq 2$. Then,

$$f_n^0(k; s+1, t) - f_n^0(k; s, t) = f_{n-1}(0; s+1, t) - f_{n-1}(0; s, t)$$

and from the inductive hypothesis

$$f_n^0(k; s+1, t) - f_n^0(k; s, t) \leq 0. \tag{3.13}$$

Also,

$$f_n^1(k;s+1,t)-f_n^1(k;s,t)=\left(\frac{s}{s+t}-\frac{s+1}{s+t+1}\right)(k+1)B$$
$$+\frac{s+1}{s+t+1}f_{n-1}(0;s+2,t)-\frac{s}{s+t}f_{n-1}(0;s+1,t)$$
$$+\frac{t}{s+t+1}f_{n-1}(k+1;s+1,t+1)-\frac{t}{s+t}f_{n-1}(k+1;s,t+1).$$

From the inductive hypothesis that $f_{n-1}(0;s+2,t)\le f_{n-1}(0;s+1,t)$ and $f_{n-1}(k+1;s+1,t+1)\le f_{n-1}(k+1;s,t+1)$,

$$f_n^1(k;s+1,t)-f_n^1(k;s,t)\le\frac{t}{(s+t)(s+t+1)}\{-(k+1)B+f_{n-1}(0;s+1,t)$$
$$-f_{n-1}(k+1;s,t+1)\}.$$

Also, from the inductive hypothesis that $f_{n-1}(0;s+1,t)\le f_{n-1}(0;s,t)$ and $f_{n-1}(k+1;s,t+1)\ge f_{n-1}(k+1;s,t)$,

$$f_n^1(k;s+1,t)-f_n^1(k;s,t)\le\frac{t}{(s+t)(s+t+1)}\{-(k+1)B+f_{n-1}(0;s,t)$$
$$-f_{n-1}(k+1;s,t)\}.$$

It is easily derived from (3.8) and this inequality that

$$f_n^1(k;s+1,t)-f_n^1(k;s,t)\le 0. \tag{3.14}$$

Therefore, (3.11) is immediate from (3.13) and (3.14). The proof of (3.12) is in the similar manner.□

4. Some relations with bandit problems.

In the classical Bernoulli two-armed bandit problem with one arm known and a finite time horizon, there are two arms, a_0 and a_1, and if a_0 is pulled, the value 1 or 0 is obtained from the Bernoulli distribution with parameter p or $1-p$ respectively and if a_1 is pulled, a value 1 or 0 is obtained from the Bernoulli distribution with parameter q or $1-q$, respectively. Although the true value of q is known a priori, the true value of p is unknown and there is the prior information that p has the Beta distribution with parameters s and t as the conjugate prior distribution. There are n stages remaining and at any stage we can select one of two arms a_0 and a_1 and pull it. After observing its value at each stage, we can select one of two arms at the next stage. The objective is to maximize the total expected reward. If the number of remaining stages is included in the state vector, then the state is denoted by $(n;s,t)$ when there are n stages remaining and the current prior knowledge is (s,t).

In state $(n;s,t)$, if a_0 is selected and the value 1 is obtained, the state will make a transition to $(n-1;s+1,t)$. If a_0 is selected in state $(n;s,t)$ and the value 0 is obtained, the next state will be $(n-1;s,t+1)$. If a_1 is selected in state $(n;s,t)$, the next state will be $(n-1;s,t)$ whichever value is obtained.

For this kind of the two-armed bandit problem, one of the interesting properties is whether the stay-on-a-winner rule holds or not. The stay-on-a-winner rule is the rule to pull the same arm if the result of a pull of one arm in the last stage is 1, that is, 'win'. In the sequential decision problem considered in this paper, the state is denoted by a 4-tuple

$(n, k; s, t)$ and it makes a transition from $(n, k; s, t)$ into $(n-1, 0; s, t)$ if a_0 is selected and in this case to select a_0 in state $(n-1, 0; s, t)$ seems to be less preferable than that in state $(n, k; s, t)$. If a_1 is selected in state $(n, k; s, t)$ and no accident occurs, the state will make a transition into $(n-1, k+1; s, t+1)$ and a_1 seems not to be preferable than that in state $(n, k; s, t)$ because the number of unbackuped files increases and therefore the risk becomes larger than before. Also, if a_1 is selected in state $(n, k; s, t)$ and an accident occurs, the state will make a transition into $(n-1, 0; s+1, t)$ and a_1 seems to be more preferable than that in state $(n, k; s, t)$ when k is large. Whether the properties stated in this paragraph hold or not is the open question.

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