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APPRAISING THE EFFECTIVENESS OF GP IN INCORPORATING THE DECISION MAKER(DM)'S PREFERENCES

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Abstract There are several difficulties with GP. This paper is an appraisement to the effectiveness of GP in incorporating DM's preferences and explains some fundamental relationships among the different approaches of incorporating. Since each approach has its own way and properties, we integrate the approaches by a unique classification. We explain why and how inappropriate approximation of nonlinear functions may lead to missing information about DM's preferences and how we can minimize the lost information in GP. Finally, an effective and interactive procedure with a numerical example is suggested in order to help analysts in defining the DM's preference functions more effectively.

1. Introduction

In order to solve any decision problem, we have to identify an opportunity set and preference functions. Most of complex decision problems involve multiple conflicting objectives and often no dominant alternative will exist that is better than all other alternatives in terms of all of these objectives. In this case, our problem is one of value tradeoffs that requires the subjective judgement of the DM. In the case of decision problems with multiple and conflicting objectives, in order to solve a system of simultaneous equations and to do a well trade-off analysis, Goal Programming (GP) is applied. Hannan[3] places GP position between multi-objective mathematical programming, where DM's preferences or tradeoff ratios for different objectives are not asked, and multi-attribute utility theory, where the values of various parameters are determined. There are some difficulties and criticisms on the use of GP models such as dominance, inferiority, and inefficiency in GP solutions, naive relative weighting and prioritization in GP models, incommonsurability and redundancy. One of the important difficulties with GP is that GP can not incorporate DM's preferences effectively. Since in GP relative weights for different goals are often viewed as a type of utility function of DM, Rosenthal[10] has argued that relative weights will almost never reflect the true decision making environment.

This paper is an attempt to answer some important questions such as:

(A) Is GP able to reflect the DM's utility function appropriately?

(B) What are the most important limitations and difficulties with GP in incorporating DM's preferences?

(C) How can we remove or relax these limitations and difficulties as much as possible?

In section 2 we will answer to the first and the second questions by explaining the properties and the restrictions of each kind of GP in incorporating DM's preferences. In section 3 we will answer to the third question by applying the interactive piecewise linearzed general value function method. In order to help analysts in determining the DM's preferences, that is a difficult and an important step in modeling process, we will introduce an effective and interactive procedure to define DM's preferences for different criteria. A numerical example is described for proposed procedure. Finally, conclusions and final remarks are provided.

2. Literature Review

One of the important difficulties with GP is that it can not reflect DM's preferences appropriately. In order to determine the tradeoff ratios for various objectives, several methods have been suggested. Charnes and Cooper[1] suggest the interval methodology, Gass[4] proposes the normalized vector method based on the AHP of Saaty, O'Leary[8] proposes the conjoint analysis, some researchers [2,11]suggest to utilize the concept of the fuzzy set and membership function, and Takeda and Yu[13] provide a well discussion on the pairwise comparisons based on the Habitual Domain Theory (HDT).

Schniederjans[12] shows that all the GP models can be decomposed into *preemptive lexicographic* GP model, in which each of the functionals or goals are given by a separate priority and the model has no weights but only a preemptive ranking for each of goals, and the *nonpreepmtive weighted* GP model, in which the relative weights to goals in the same priority level will be assessed, ranked, and incorporated into the objective function of GP models. These two extremes are not comprehensive for making all kinds of GP models because they can explain GP models with goals that have constant tradeoff ratios and then can not explain nonlinear models effectively. What if the DM's values for different goals change over the different obtainable level of goals? In nonlinear cases, linear GP models are not effective in incorporating preferences and reflecting the real decision environment unless they compensate the lost information by introducing several linear segments interactively and incorporating them into the objective function of GP models. Value Function method and Fuzzy method are two important methods in this area.

Hannan[3] suggests that as a first step the analyst should develop a *value function* that is actually representative of the DM's values. A function $v(X)$ is said to be a value function representing the DM's preference structure provided that $X' \sim X" \Leftrightarrow v(X') = v(X")$ and $X' > X" \Leftrightarrow v(X') > v(X")$, we find action $a \in A$ to maximize $v[X(a)]$ where X is alternative vector. If $(x_1, x_2, \ldots, x_n)$ is a point or an alternative, $v(x_1, x_2, \ldots, x_n) \succeq v(x'_1, x'_2, \ldots, x'_n) \Leftrightarrow (x_1, x_2, \ldots, x_n) \succeq (x'_1, x'_2, \ldots, x'_n)$ where $\succeq$ reads preferred or indifferent to. Also we can find some functions f with a simple form such that $TV(x_1, x_2, \ldots, x_n) = f[v(x_1), v(x_2), \ldots, v(x_n)]$ where in complex multiattribute problems v_i designates a value function over the single attribute X_i. We assume that each attribute is positively oriented, the more of any component the better, given any fixed level of the other components. To hold all attributes fixed and look at the substitution rates as a function of a given amount of an attribute, we can realize the marginal rates of substitution for each attribute. The marginal rate of substitution between X and Y depends on y not on x if and only if there is a value function v of the form $v(x, y) = x + v(y)$ over attribute Y. The additive value function is about as simple as we can find. A preference structure is additive and therefore has an associated value function of the form $v(x, y) = v_X(x) + v_Y(y)$ where v_X and v_Y are value functions. A value function v may be expressed in an additive form $v(x, y, z) = v_X(x) + v_Y(y) + v_Z(z)$ where v_X, v_Y, and v_Z are single-attribute value functions, if and only if (X, Y) is Preferentially Independent (PI) of Z, (X, Z) is (PI) of Y, and (Y, Z) is (PI) of X. In other words, each pair of attributes must be (PI) of the remaining attributes. Given attributes $X_1, X_2, \ldots, X_n, n \geq 3$ an additive value function $v(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} v_i(x_i)$ where v_i is a value function over X_i exists if and only if the attributes are mutually (PI). If each pair of attributes is (PI) of its complement, the attributes are pairwise (PI). Considering these assumptions, we can easily make $v(x_1, x_2, \ldots, x_n) = \sum_{i=1}^{n} v_i(x_i)$ by the component value functions[6].

A calculation in GP consists of minimizing the gap between the attained level of each objective and the goal for this objective. Defining g_i as a target level of the ith goal ($i = 1, 2, \ldots, m$) and a_{ij} as the contribution of the jth variable ($j = 1, 2, \ldots, n$) in the ith goal, for each solution $x = (x_1, x_2, \ldots, x_n) \in (X)$, where (X) denotes feasible solution set, we can measure the deviation by $d_i = |\sum_{j=1}^{n} a_{ij}x_j - g_i|$. Since less deviation being prefer to more (or equivalently more goal achievement being prefer to less), a component value function $v(d_i)$ for the ith deviational variable in terms of alternative $(x_1, x_2, \ldots, x_n)$, should be a decreasing function of d_i (or increasing function of goal achievement g). In mathematical terms we have $\frac{\partial v(d_i)}{\partial d_i} < 0$ or $\frac{\partial v(g_i)}{\partial g_i} > 0$. $v(d_i) \in [0, 1]$ that its values are determined by DM where $v(d_i) = 1$ if $d_i = 0$, and $v(d_i) = 0$ if d_i takes its maximum value. Equivalently, $v(g_i) \in [0, 1], v(g_i) = 1$ if g_i takes its maximum value, and $v(g_i) = 0$ if g_i takes its minimum value. v_i may be in nonlinear form. In this case, the value function approach approximates nonlinear functions with several piecewise linear functions. According to the value function method, in order to define DM's value function in Multiple Criteria Decision Making (MCDM) and specially in GP, we have to define multi-valued functions for multiple criteria or goals. A general definition of value function for linear GP models with m goals can be expressed by

$$TV(d) = \sum_{i=1}^{m} v(d_i) = v(d_1^+, d_2^+, \ldots, d_m^+, d_1^-, d_2^-, \ldots, d_m^-)$$

or equivalently,

$$TV(g) = \sum_{i=1}^{m} v(g_i) = v(g_1, g_2, \ldots, g_m)$$

where $TV(d)$ and $TV(g)$ are the value functions, $v(d_i)$ and $v(g_i)$ are the component value functions in terms of deviational variable and obtained level of goal, respectively. Some forms of decreasing functions in terms of deviational variable and equivalently increasing functions in terms of goal achievement level are shown by Figure 1.

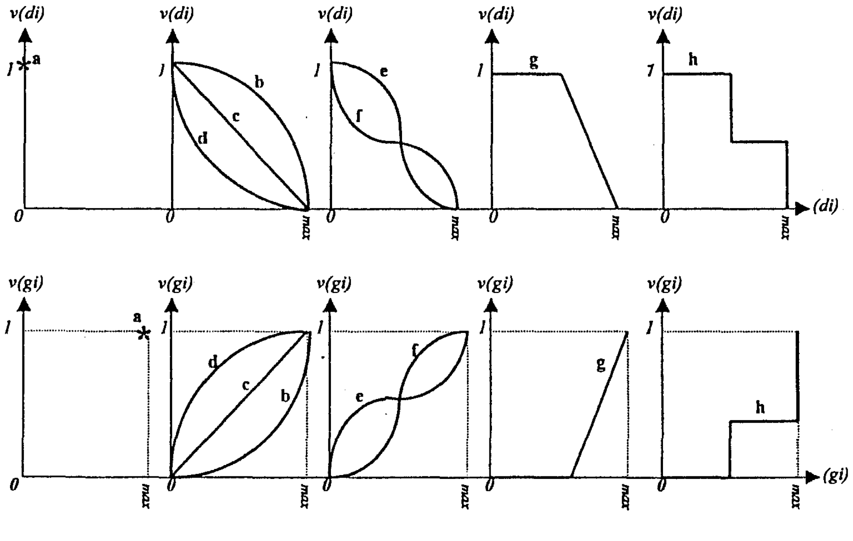

Figure 1: Some forms of decreasing and increasing functions

For some nonlinear functions, such as quadratic functions, there are efficient algorithms and softwares to optimize a quadratic objective function subject to some linear constraints. For others, higher order functions, we should use of nonlinear GP or should accurately approximate nonlinear functions by some piecewise linear segments and then use of linear GP. Inappropriate approximations will decrease the effectiveness of a decision making because of losing the important information about DM's preferences. The pioneer paper of Pratt[9], for example, explains that investor's behavior may be well expressed only by nonlinear functions in mean-variance method and if we approximate investor's nonlinear functions inaccurately, we ignore the important information about the investor's real preferences and consequently can not express and incorporate DM's preferences appropriately.

According to Hannan's suggestion and also well known generalized criterion method of Promethée, Martel and Aouni[7] propose an approach in order that the DM can build his preference functions along with the goals he has fixed for each objective. Rather than trying to establish utility functions for different values of each objective and translate them into objective function of GP as Hannan suggests, they refer to the idea of generalized criterion method. This idea not only leads to differences of performance between two actions on a given criterion, but also leads to consideration of value functions expressing the density of DM's preferences in a way which is easy for him to understand. They suggest that preference functions may be different for negative and positive deviational variables and for the different objectives. They forget to consider the effects of time horizon in changing DM's preference functions in multiperiod models. For example, in multiperiod portfolio models, the preference of investor for risk may change with the time horizon of portfolio decision making. Gunthorpe and Levy[5] found that increasing the horizon, investors prefer to hold more safety securities in their portfolios.

Sometimes DM may not be able to express his target levels for different goals precisely. In these cases, *Fuzzy* or imprecise goals of the DM may be incorporated into a standard GP formulation and new problem can be solved by using the properties of piecewise linear continuous functions and GP deviational variables[2,11]. Interactive methodology for solving the fuzzy GP problem is as follow:

(A) Elicit a membership function for $f_i(z_i)$ from the DM for each of objective functions $z_i, i = 1, 2, \ldots, k$. We can quantify goals "somewhat larger than" or "less than" by eliciting a membership function or by scaling it between 0 (no degree of membership) and 1 (complete membership).

(B) Connect the points in the discrete membership function with line segments to obtain Interpolated Membership Function (IMF), that is a piecewise linear and continuous function.

(C) Convert each membership function $f_i(z_i)$ into $f_i(z_i) = \sum_{i=1}^{N} \alpha_i |z_i - g_i| + \beta z_i + \gamma$ where $\alpha = \frac{t_{j+1} - t_j}{2}$, $\beta = \frac{t_{N+1} - t_1}{2}$, $\gamma = \frac{s_{j+1} - s_1}{2}$ and for each segment $g_{r-1} \leq z_i \leq g_r$ we have $f_i(z_i) = t_r z_i + s_r$. t_r is the slope and s_r is the y-intercept for the section of the curve initiated at g_{r-1} and terminated at g_r. z_i represents an objective of the form $z_i = \sum_{j=1}^{n} C_j^i x_j$, $f_i(z_i)$ values are grade of membership, $0 \leq f_i(z_i) \leq 1$ for all values of z_i. Defining z_i^* and z_i^l as the maximum and minimum possible values for an objective function, we can associate $f_i(z_i) = 1$ for $z_i \geq z_i^*$, $f_i(z_i) = 1 - \frac{z_i^* - z_i}{z_i^* - z_i^l}$ for $z_i^l \leq z_i \leq z_i^*$, and $f_i(z_i) = 0$ for $z_i \leq z_i^l (i = 1, 2, \ldots, k)$.

(D) Determine the objective function:(1) DM may use goals and preemptive priorities with different order of magnitude,(2) DM may weight the various goals and assume they are all of the same order of magnitude.

The other important difficulty is that many of DM's do not obey all the rationality when faced with a series of complicated and multiple criteria choice situations because DM encounter aspects of the problem that were not of concern to them in simple situations and this kind of behavior may create some difficulties in defining DM's value functions. Equipped with literature, in the next section we will provide an effective and interactive procedure in the manner that minimizes the difficulty of defining DM' preferences.

3. An Effective Procedure for Defining DM's Preferences

In order to make optimal decisions DM should be informed of opportunity set of his decision problem. The best opportunity could be extracted from the set by defining DM preferences. Several approaches and models are provided for different kinds of GP models. The approaches are different in terms of the form of objective function and each of them, except general value function approach, is efficient and effective for special models. The most important work of an analyst is to determine the best approach for a given GP problem. A multiobjective GP problem may be in (1) lexicographic form, DM may use goals and preemptive priorities with different order of magnitude, (2) non-preemptive weighted form, DM may weight the various goals and assume they are all of the same order of magnitude, (3) preemptive weighted form, consists of some goals with different order of magnitude and some with the same order of magnitude, (4) or in general value function form.

As a first step, analyst should separate three kinds of goals, absolute prior goals with no substitution rates, goals with constant rate of substitution, and goals with variable rate of substitution. Then, if all goals are in absolute prior form, the lexicographic form is valid. If all goals have constant tradeoff ratios, the non-preemptive weighted form is valid. If some goals have constant tradeoff ratios and some are in absolute prior form, the preemptive weighted form is valid. And if there are some goals with variable rate of substitution, we may express each of them with linear segments and use of non-preemptive weighted form or it may be better to apply the general value function. The following interactive procedure will help analysts in determining the DM's preferences more effectively:

(I) what criteria $\{C_1, C_2, \ldots, C_m\}$ are included in MCDM problem?

(II) when the pairwise comparison is possible and desirable for DM, create list of all possible pairs of criteria $\{C_1 <=> C_2, C_1 <=> C_3, \ldots, C_1 <=> C_m, \ldots, C_{m-1} <=> C_m\}$ and provide it to the DM for the comparison. Rewrite the results in the form of $C_i > C_j$ where $i \neq j$. Finally, rank criteria from the most important criterion to the inferior one in terms of times that a criteria has been in greater side of comparisons. After ranking go to the next step.

(III) which criteria have no substitution or have absolute priority to the other criteria? In other words, are there criteria with infinite substitution rate related to each of the other criteria? Extract and sort them according to their importance sequentially in the form of $(P_1 >>> P_2 >>> \ldots >>> P_L)$, where the symbol of $>>>$ in $P_i >>> P_j$ means P_i is very very bigger than P_j. P is the symbol of absolute priority so that devotes an absolute priority to a criterion in terms of the other inferior criteria. If $L = m$, we have$P_1 >>> P_2 >>> \ldots >>> P_m$ or we have a *lexicographic GP model.* If $L < m$ go to the next step.

(IV) if all $[m - L]$ criteria are quantitatively comparable and it is desirable for DM to determine constant tradeoff ratios, arrange all pairs of remaining criteria from the easiest pair to compare to the most difficult pair. Starting with the easiest pair we can ask DM, is criterion i at least t times more important than criterion j? If not, is criterion i at least $t - s$ times more important than criterion j (for $s < t$)? and so on.

(V) if the answer to the first question in step 4 is yes, we can ask DM, "is criterion i at least $t + q$ times more important than criterion j?" and so on. By this way, we can define $W_1 > W_2 > \ldots > W_f$ by pairwise comparison methods. If $L = 0$ and $f = m$, we have $W_1 > W_2 > \ldots > W_m$ or a *nonpreemptive weighted GP model*, and if $L > 0$, $f > 0$,and $L + f = m$, we have *preemptive weighted GP model.*

(VI) if $L > 0$, $f > 0$ and $L + f < m$, there are some criteria that are not absolutely prior and have not constant tradeoff ratios. When the expression of priority weights is difficult for DM and the interpretation of the coefficient remains complex and ambiguous, we can determine DM's preference over the range of deviation from the target level and the effective approach is *general criterion method* (GCM) in which we define a value function for each criterion and the objective is to maximize DM's total goal achievement level. The difference between fuzzy GP and this method is not in the resulting mathematical formulation but essentially with respect to the philosophy that underlies the DM's input. According to GCM, for each objective i and for each pair of actions $(x, y) \in X^2$, we can associate a function $P_i(x, y)$ measuring the DM's preference intensity for x over y in order to have

$P_i(x, y) = 0$ for indifference, $f_i(x) \cong f_i(y)$;
$P_i(x, y) \cong 0$ for week preference, $f_i(x) \succ f_i(y)$;
$P_i(x, y) \cong 1$ for strong preference, $f_i(x) \succ\succ f_i(y)$;
$P_i(x, y) = 1$ for strict preference, $f_i(x) \succ\succ\succ f_i(y)$;

where $f_i(x) = \sum_{j=1}^{n} a_{ij}x_j$ and $f_i(y) = \sum_{j=1}^{n} a_{ij}y_j$. Defining $d_i = f_i(x) - f_i(y)$, as the difference in performance between the action x, y in relation to criterion i, this performance function $P_i(x, y)$ can be defined by a criterion or value function noted by $\bar{F}(d_i)$. $\bar{F}(d_i)$ expressing the intensity of DM's preferences in such a way that it is easy for him to understand. GCM form will be as $MaxZ = \sum_{i=1}^{m}(F_i^+ d_i^+ + F_i^- d_i^-)$. The value of Z then could be translated in terms of a realization percentage of goal fixed by the DM and ranges from 0 to m where m is number of criteria or goals and the closer Z is to the value m, the more globally satisfied the DM.

In order to extract DM's value function by some sub-value functions, considering GCM method, we can perform the following process:

(I) calculate individual minimum and maximum of each of the objective functions under the given constraints (f_i^{min}, f_i^{max}) and determine the range of deviational variable for each goal.

(II) provide to the DM different types of functions so that the DM can select functions for different objectives from among them in an interactive and subjective manner. A criterion function may be in linear, exponential, hyperbolic, hyperbolic inverse, piecewise linear form[11]. Also Promethée proposed six types of criterion function each one corresponding to one type of criterion from which DM can easily find the functions corresponding to his preferences for each objective[7].

(III) Determine the parameter values through an interaction with the DM.

This interactive, directive, and suggestive procedure as an framework can help analysts in determining DM's preferences. The flow chart of the procedure is as Figure 2:

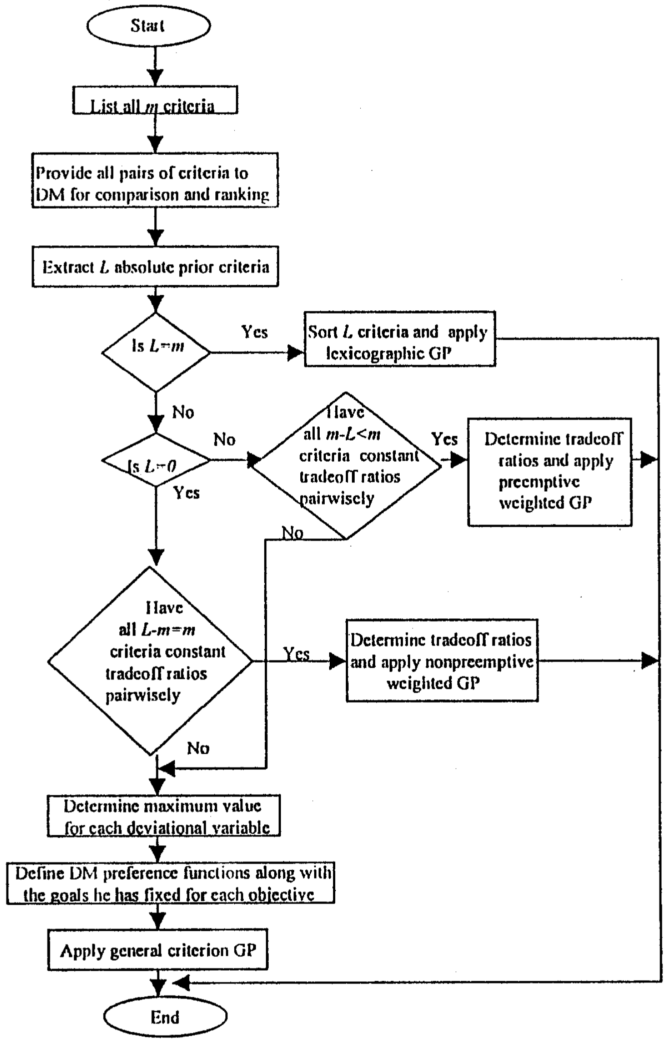

Figure 2: An interactive and directive procedure for determining DM's preferences

The following numerical example demonstrates the procedure more clearly.

4. A Numerical Example

In order to explain the proposed procedure, consider a production decision problem that is illustrated in reference[7]. The first question from DM is "what criteria $\{C_1, C_2, \ldots, C_m\}$ are included in MCDM problem?" Since DM considers 5 criteria, then the answer will be $m = 5$ or $\{C_1, C_2, C_3, C_4, C_5\}$.

The second question from DM is that "is the pairwise comparison possible and desirable for you?" If the answer is yes, we create list of all possible pairs of criteria $\{C_1 <=> C_2, C_1 <=> C_3, C_1 <=> C_4, C_1 <=> C_5 \ldots, C_4 <=> C_5\}$ and provide it to the DM to compare. For example, one possible answer may be as $\{C_1 > C_2, C_1 > C_3, C_1 < C_4, C_1 < C_5, C_2 > C_3, C_2 < C_4, C_2 < C_5, C_3 < C_4, C_3 < C_5, C_4 > C_5\}$. Rewrite the results in the form of $C_i > C_j$ where $i \neq j$ as $\{C_1 > C_2, C_1 > C_3, C_4 > C_1, C_5 > C_1, C_3 > C_2, C_4 > C_2, C_5 > C_2, C_4 > C_3, C_5 > C_3, C_4 > C_5\}$. C_4 is 4 times in greater side, C_5 is 3 times in greater side, C_1 is 2 times in greater side, C_3 is 1 time in greater sidein greater side, and finally C_2 is 0 time in greater side. Then, we can rank criteria from the most important criterion to the inferior one as $\{C_4, C_5, C_1, C_3, C_2\}$. After ranking go to the next step.

Which criteria have no substitution or have absolute priority to the other criteria? If all, then we have $P_1 >>> P_2 >>> P_3 >>> P_4 >>> P_L$. When $L = m$, $P_1 >>> P_2 >>> P_3 >>> P_4 >>> P_5$ or we have a *lexicographic GP model* as $C_4 >>> C_5 >>> C_1 >>> C_3 >>> C_2$. If $L = 0$ go to the next step.

$L = 0$ means that all criteria are in the same priority level. When all $[m - L]$ criteria are quantitatively comparable and it is desirable for DM to determine constant tradeoff ratios, arrange all pairs of criteria from the easiest pair to compare to the most difficult pair. Starting with the easiest pair we can ask DM, is criterion i at least, for example, 5 times more important than criterion j? If not, is criterion i at least $5-1 = 4$ times more important than criterion j? and so on. If criterion i was at least 5 times more important than criterion j, we could ask DM, is criterion i at least $5 + 1 = 6$ times more important than criterion j? and so on. From the result of comparisons, we can create a reciprocal matrix and apply some well known methods such as Eigenvector Method (EM) to calculate the normalized priority weight vector. Suppose that C_4 is 2 times more important than C_5, 2 times more important than C_1, and 9 times more important than C_2. C_5 is 9 times more important than C_3, and C_3 is 2 times more important than C_2. Applying (EM), the normalized priority weight vector is $W = (.420, .308, .191, .048, .034)$. By this way, we can define the objective function of nonpreemptive weighted GP model. If $L = 0$ and $m - L = f = m$, we have $W_1 > W_2 > \ldots > W_5$ or a *nonpreemptive weighted GP model.*

When $L > 0$, and all $m - L$ criteria are in the same priority level, and also it is possible and desirable for DM to determine constant tradeoff ratios for all same priority level criteria, we have a *preemptive weighted GP model.* Suppose, for example, only C_4 has absolutely priority to the other criteria and C_5 is 5 times more important than C_1, 7 times more important than C_3, and 3 times more important than C_2. C_1 is 6 times more important than C_3, and 6 times more important than C_2. Also C_3 is 5 times more important than C_2. Applying geometric mean method (GMM), the priority weight vector is $W = (.557, .285, .102, .056)$. Finally, we can easily define the objective function of preemptive weighted GP model.

If $L > 0, f > 0$ and $L + f < m$, there are some criteria that are not absolutely prior and have not constant tradeoff ratios. In this case and also when the pairwise comparision is not possible or desirable (as stated in step 2), we can determine DM's preference over the range of deviation from the target level and the effective approach is *general criterion method* (GCM) in which we define a value function for each criterion and the objective is to maximize DM's total goal achievement level. Suppose that criterion functions for production problem are as Figure 3:

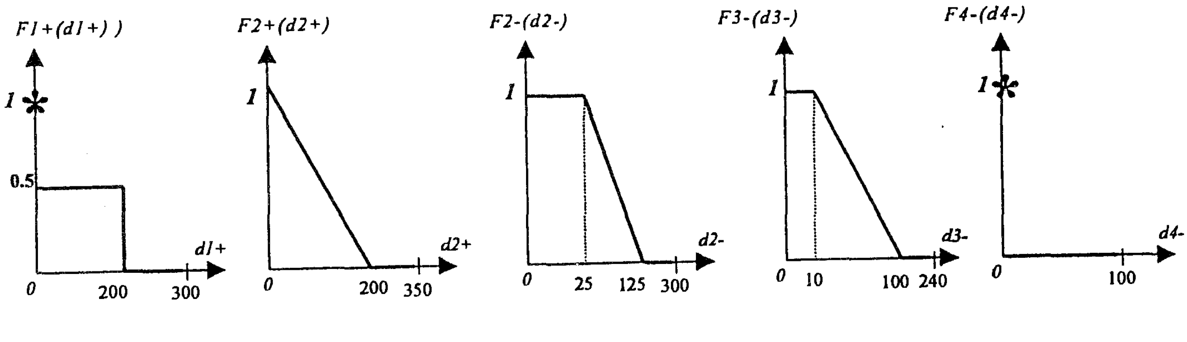

Figure 3: Criterion functions for production problem

$F_1^+(d_1^+) = 1$ if $(d_1^+) \leq 0$, $F_1^+(d_1^+) = 0.5$ if $0 < (d_1^+) \leq 200$, $F_1^+(d_1^+) = 0$ if $200 < (d_1^+) \leq 300$. $F_2^+(d_2^+) = 1 - 0.0005d_2^+$ if $0 < (d_2^+) \leq 200$, $F_2^+(d_2^+) = 0$ if $200 < (d_2^+) \leq 350$. $F_2^-(d_2^-) = 1$ if $0 \leq (d_2^-) \leq 25$, $F_2^-(d_2^-) = 1.25 - 0.01d_2^-$ if $25 < (d_2^-) \leq 125$, $F_2^-(d_2^-) = 0$ if $125 < (d_2^-) \leq 300$. $F_3^-(d_3^-) = 1$ if $0 \leq (d_3^-) \leq 10$, $F_3^-(d_3^-) = 1.11 - 0.011d_3^-$ if $10 < (d_3^-) \leq 100$, $F_3^-(d_3^-) = 0$ if $100 < (d_3^-) \leq 240$. $F_4^-(d_4^-) = 1$ if $(d_4^-) = 0$, $F_4^-(d_4^-) = 0$ if $0 < (d_4^-) \leq 100$. These relations can be easily incorporate into the objective function of GP model subject to some constraints. Since we are not to explain the mathematical formulation process of these criterion functions, see reference [7] for mathematical formulation.

5. Conclusions and Final Remarks

Solving any decision problem requires to the opportunity set and DM's preferences. In this paper, we assessed different GP approaches in terms of incorporating DM's preference. Since sometimes DM have some nonlinear preference function, approximating a nonlinear preference functions inappropriately may lead to oversimplifying the reality of the problem, losing the important information about the DM's preferences, and decreasing the effectiveness of a decision making. By applying generalized criterion method and approximating nonlinear preference function by a function with several pieces of linear segments we can relax and minimize DM confusion and make it is to understand. Since defining DM's preferences for different objectives regards a difficult and important step in modeling process, we proposed an interactive and suggestive systematic procedure in order to maximize the effectiveness of GP models in incorporating DM'preferences. To demonstrate the procedure, finally a numerical example is provided.

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