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A MUTUAL OVERFLOW SYSTEM WITH SIMULTANEOUS OCCUPATION OF RESOURCES

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Abstract In this paper, we consider a special mutual overflow system for the analysis of RTNR (Real Time Network Routing) dynamic routing networks. The system has three multiserver service facilities and no waiting room is available. The first-offered traffic to a facility is generated to form an independently and identically distributed Poisson process. If all the servers of a facility are busy, the blocking occurs. Blocked customer overflows to the other facilities and service is required from both of the rest facilities simultaneously. If the service can be provided immediately, then two servers that belong to their respective facility are assigned to the customer and start or finish service at the same time. Otherwise, the overflows will be cleared from the system. For such a system, we describe an algorithm to compute exactly the joint state probabilities by considering the server reservation for first-offered traffic. Following this result, we give a series of steady-state performance measures including blocking probability, carried traffic, utilization of facilities and so on. Numerical examples will also be discussed for understanding of the behavior of load balance mechanism in such a system and with simultaneously to show the effectiveness of the presented performance evaluation method.

1. Introduction

Mutual overflow system is a kind of queuing model that is mainly resulted from dynamic routing telecommunications networks for load balance. In such a system, overflow traffic caused by local overload is allowed to be carried through other trunk groups or links. The reviews of both theoretical and practical orientated studies for such a system have been included in the literature [1]-[6] that can be mainly summarized up to three basic problems as follows,

(1) blocking in a dynamic integrated services network;
(2) performance of load balance with traffic overflow;
(3) service requirement from several facilities simultaneously.

Among these problems, Dziong and Roberts [7], Tsang and Ross [8] have given an efficient convolution algorithm for the exact calculation of state probabilities of problem (1). The recent research result on overflow system is by El-Taha and Heath [9], where they considered a closed queuing network with some primary servers, some secondary servers and limited buffer size. Their main result is an efficient algorithm for the exact calculation of joint state probabilities in such a system.

We are interested in performance analysis issues on RTNR (Real Time Network Routing) networks. Different from other dynamic routing approach, the distinguished feature of RTNR is its two-link routing restriction, where tandem routings consisted of more than two links are forbidden. Until now, although some very important parameters such as the level of trunk reservation have been put into practical uses, the theoretical studies for getting these parameters have to be limited to simulation or to be approximated like "link independence and Poisson overflow traffic etc.", due to some inherent mathematical difficulties involved for above problem (2) and (3) [4].

Particularly, the difficulty to analyze problem (3) is that there is a competition among the resources. That is not only from other demands for circuits on the same path, but also from demands for different circuits that use only some of the same facilities. Moreover, the customers, who typically require the simultaneous occupation of limited resources associated with several different facilities, are from overflow. The literature on this problem is summarized in [10] and the latest discussions were by Whitt [9], and Lu etc. [5]. Unfortunately, some approximations were still involved in these studies.

In this paper, we consider a special mutual overflow system for the analysis of RTNR networks. In section 2, as a special case of El-Taha and Heath's model with k=0 [9], an algorithm for exact calculation of joint state probabilities of a conventional overflow system is first presented. In section 3, we describe an algorithm to compute exactly the joint state probabilities of a mutual overflow system. Following this result, in section 4, we give a series of steady-state performance measures including blocking probability, carried traffic, utilization of facilities and so on. In section 5, Numerical examples will also be discussed for understanding of the behavior of load balance mechanism in such a system and with simultaneously to show the effectiveness of the presented performance evaluation method.

2. System Model and Assumptions

2.1 System Model

We consider a general queuing system with three multiserver service facilities named facility m, $(m=1,2,3)$. Facility m consists of $N_m \geq 1$ exponential servers with mean $1/\mu$ and no waiting room is available.

The first-offered traffic to facility m is generated to form an independently and identically distributed Poisson process with parameter λ_m (m=1,2,3). If all the servers of facility m are busy, the blocking occurs. Blocked customers of the first-offered traffic overflow to the other facilities and service is required from the rest two facilities simultaneously. If the service can be provided immediately, then two servers that belong to their respective facility are assigned to a customer and start or finish the service at the same time. Otherwise, the blocked customer will be cleared from the system. To protect against spread of overload, server reservation for first-offered traffic is allowed. In this case, each facility has a permission threshold of available servers for the overflow traffic.

Let r_m ($0 \leq r_m \leq N_m$) be the threshold of server reservation in facility m that represents the maximum number of available servers for overflow traffic in the facility. To serve the overflow from facility m (m=1,2,3), we consider three virtual facilities with maxim capacity of M_m servers (m=1,2,3) respectively, (see Fig.1) where $M_1 = \min\{r_2, r_3\}$, $M_2 = \min\{r_1, r_3\}$, and $M_3 = \min\{r_1, r_2\}$. Then, we define the system state in equilibrium by the number of busy servers in each facility as $X = (x_1, x_2, x_3)$, where $0 \leq x_m \leq N_m$ $(m=1,2,3)$.

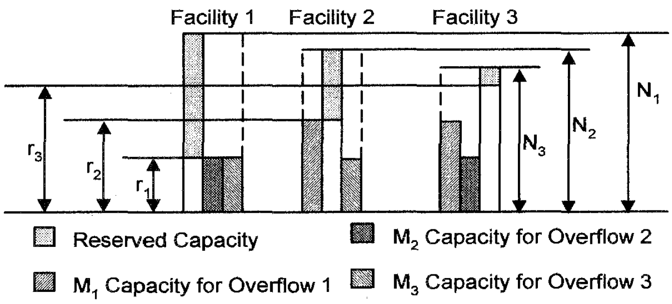
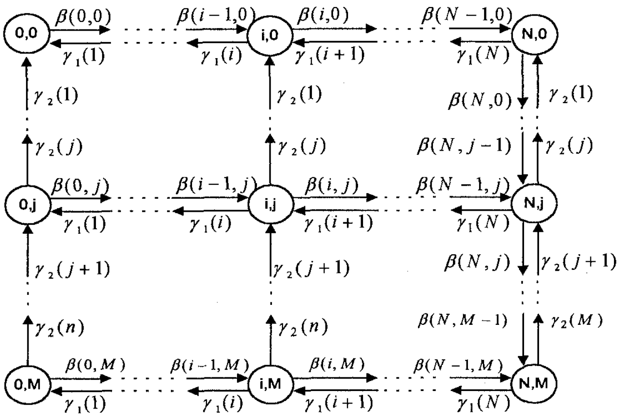


Fig. 1 Explanation of partial sharing.

2.2 Conventional Overflow Model

To deal with above queuing model, we first consider a conventional overflow system as a special case of El-Taha-Herath's model with $k \equiv 0$ [8]. The system consists of N primary servers and M secondary servers. An arrival of a Poisson stream with rate λ first joins a server of primary group and receives a service, if available, otherwise it overflows to the secondary group. Service rates in primary and secondary groups are independent exponentially distributed with mean $1/\mu$, respectively.

We define the system state to be an ordered pair (i, j), where i (i=0,1,...,N) is the number of busy primary servers, while j (j=0,1,...,M) is the number of busy secondary servers. Also denote $\pi_{i,j}$ to be the steady joint probability of state (i, j). Then the state-dependent arrival and service rates are shown in Fig.2,



where

$$\beta(i,j) = \begin{cases} \lambda, & (0 \le i \le N, \quad 0 \le j < M,) \\ & \text{or} \quad (0 \le i < N, \quad j = M,) \\ 0, & \text{otherwise,} \end{cases}$$

$$\gamma_1(i) = \begin{cases} i\mu, & 0 \le i \le N, \\ 0, & \text{otherwise,} \end{cases}$$

$$\gamma_2(j) = \begin{cases} j\mu, & 0 \le j \le M, \\ 0, & \text{otherwise.} \end{cases}$$

Fig.2 State transition diagram of a conventional overflow system.

By extending the known results according to [8], we have

$$\pi_{i,j} = \sum_{k=0}^{\min\{i,M-j\}} (-1)^k A(i,j,k)\pi_{0,j+k}, \qquad 0 < i \le N, \quad 0 \le j \le M, \tag{1}$$

where

$$A(i,j,k) = \begin{cases} 1, & i = 0,\ k = 0, \\ \gamma_1(i)^{-1}\big[\alpha(i-1,j)A(i-1,j,k) - \beta(i-2,j)A(i-2,j,k) \\ \qquad + \gamma_2(j+1)A(i-1,j+1,k-1)\big], \\ \qquad 0 \le i \le N, 0 \le j \le M, 0 \le k \le \min\{i, M-j\}, \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

We then give the following iterative algorithm to exactly calculate the state probabilities of a conventional overflow system [11].

[Algorithm 1] For a conventional overflow system, its steady state probability is

$$\pi_{i,j} = q_{i,j}\pi_{N,M}, \qquad 0 \le i \le N, \quad 0 \le j \le M, \tag{3}$$

where the unnormalized probability $q_{i,j}$ can be obtained by following Eqs. (4)-(7). The flow chart of an algorithm to get $q_{i,j}$ is given in Fig. 3.

In this algorithm, we described $\pi_{i,j}$ as an expression only involving the form of $\pi_{N,M}$. $\pi_{N,M}$ is the probability that the overall system is congested.

$$q_{i,j} = \begin{cases} \dfrac{A(i,M,0)}{A(N,M,0)}, & 0 \le i \le N, \quad j = M, \qquad (4) \\ \beta(N,j)^{-1}\big[\alpha(N,j+1)q_{N,j+1} \\ \quad - \beta(N-1,j+1)q_{N-1,j+1} & i = N, \\ \quad - \gamma_b(j+2)q_{N,j+2}\big] & 0 \le j < M, \qquad (5) \\ \dfrac{[q_{N,j} - \sum_{k=1}^{\min\{N,M-j\}} (-1)^k A(N,j,k)q_{0,j+k}]}{A(N,j,0)}, \\ \qquad i = 0, \quad 0 \le j < M, \qquad (6) \\ \sum_{k=0}^{\min\{i,M-j\}} (-1)^k A(i,j,k)q_{0,j+k}, \\ \qquad 0<i<N, \quad 0 \le j < M. \qquad (7) \end{cases}$$

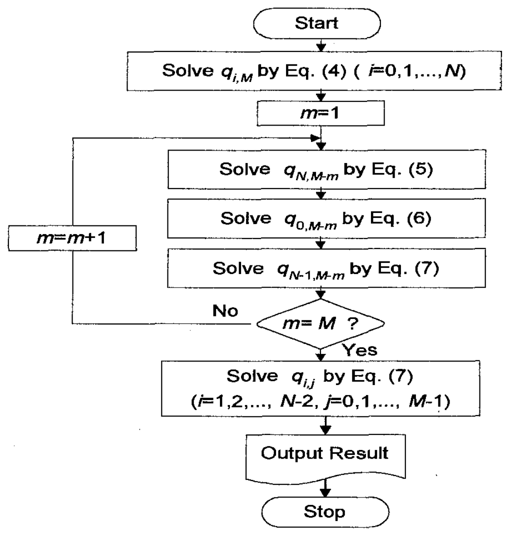


Fig.3 flow chart for getting $q_{i,j}$.

3. Joint Probabilities

3.1 Convolution Algorithm

By convolution of finite sequences, convolution algorithm is often employed to determine the exact performance parameters in telecommunication networks with partial sharing of resources involved [8]. Its main ideas are as follows.

Let $s=[s(0),s(1),...,s(N)]$ and $t=[t(0),t(1),...,t(N)]$ be two finite sequences of length N+1. Let $z=[z(0),z(1),...,z(N)]$ be the convolution of s and t, i.e.,

$$z(n)=\sum_{m=0}^{n}s(m)\cdot t(n-m), \qquad 0\le n\le N. \tag{8}$$

Denote $\otimes$ for the usual convolution operator so that $z=s\otimes t$. Then the computational effort required to obtain z from s and t through (8) is $O(N^2)$.

Now suppose that we have K finite sequences $\pi_k=[\pi_k(0),\pi_k(1),...,\pi_k(N)]$ $(k=1,2,...,K)$. Denote both

$$\pi=\pi_1\otimes\cdots\otimes\pi_K\ , \tag{9}$$

$$\pi_{(k)}=\underset{\substack{1\le l\le K\\ l\ne k}}{\otimes}\pi_l\ . \tag{10}$$

Note that π and $\pi_{(k)}$, $(k=1,2,...,K)$, are vectors of length N+1. Clearly, by successively applying Eq. (8), π can be determined in $O(KN^2)$ times. Similarly, for any given k, $\pi_{(k)}$ can be determined in $O(KN^2)$ times through (10). Thus the overall effort required to determine π and all $\pi_{(k)}$, $k=1,2,...,K$, by the above procedure is $O(K^2N^2)$. The following is an algorithm that performs this task more efficiently.

3.2 The Convolution/Deconvolution Algorithm

Note that $\pi=\pi_{(k)}\otimes\pi_k$ or more explicitly

$$\pi(n)=\sum_{m=0}^{n}\pi_k(m)\cdot\pi_{(k)}(n-m)$$

$$=\pi_k(0)\cdot\pi_{(k)}(n0)+\sum_{m=1}^{n}\pi_k(m)\cdot\pi_{(k)}(n-m), \qquad 0\le n\le N. \tag{11}$$

Thus, $\quad \pi_{(k)}=\pi(0)/\pi_k(0)$, (12)

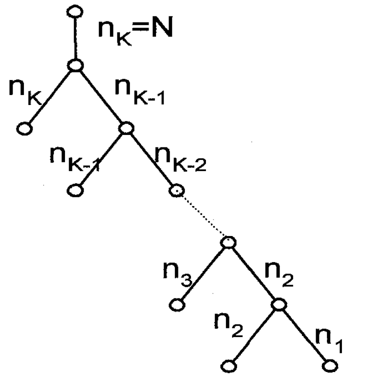
and $$\pi_{(k)}(n)=\frac{1}{\pi_k(0)}\left[\pi(n)-\sum_{m=1}^{n}\pi_k(m)\cdot\pi_{(k)}(n-m)\right], \quad n=1,2,...,N. \tag{13}$$

The effort required to deconvolve $\pi_{(k)}$ from π and π_k via Eq. (13) is $O(N^2)$. Thus, the computational effort of the algorithm is $O(KN^2)$.

3.3 Convolution Algorithm for Dynamic Partial Sharing

Now we consider a convolution algorithm for the problem of dynamic partial resource sharing. For the discussion of partial sharing problem, the relative system is usually described as a multirate tree network as in [8]. In such a modeling, the call streams are classified into K classes and offered to a system of capacity $N=n_0+\sum_{k=1}^{K}n_k$ servers. n_0 is a shared capacity that can be used by all call classes. n_k is an additional capacity called the reservation parameter for class k. Let $j_k\ge 0$ be the number of calls of class k in the system at any instant. The state space is then defined by the conditions $j_k\le n_0+n_k$, $(k=1,2,...,K)$ and $\sum_{k=1}^{K}j_k\le n_0+\sum_{k=1}^{K}n_k$. If the state probability of such a system has the product form of the marginal probability of each traffic stream, it is obtained successfully by a convolution algorithm introduced in previous subsections. For the problem of dynamic partial sharing, we give following revised algorithm.

[Theorem 1] Suppose that there are K distinguished independent traffic offered to a service facility with capacity N. The relative trunk reservation threshold for each traffic stream forms a multi-class restriction (resource occupation boundaries) as $0 \le n_1 \le n_2 \le, \dots, \le n_k \le, \dots, \le n_K$ (see Fig. 4) and the marginal probability of each traffic stream is known as $p_k(j)$, $(0 \le j \le n_k,\ 0 \le k \le K)$.



$$\begin{cases} 0 \le \sum_{i=1}^{K} j_i \le n_K, \\ \vdots \\ 0 \le \sum_{i=1}^{k} j_i \le n_k, \\ \vdots \\ 0 \le j_1 + j_2 \le n_2, \\ 0 \le j_1 \le n_1. \end{cases}$$

Fig.4. A tree network.

Let $\pi(j)$ be the joint probability of the system state with j busy servers. If $\pi(j)$ has the product form of its marginal probabilities $p_k(j)$, $(0 \le j \le n_k,\ 0 \le k \le K)$, then we can get $\pi(j)$ by a revised convolution algorithm as follows.

$$\text{Step 1:} \quad \begin{cases} q_{(K)}(j) = p_K(j), & 0 \le j \le n_K, \\ q_{(k)} = p_k \otimes q_{(k+1)}, & 0 \le j \le n_k, \quad (k = K-1, K-2, \dots, 1); \end{cases} \tag{14}$$

$$\text{Step 2:} \quad \pi(j) = \begin{cases} q_{(1)}(j), & 0 \le j \le n_1, \\ q_{(k)}(j - n_{k-1})\pi(n_{k-1}) / C_j^{n_{k-1}}, & n_{k-1} < j \le n_k, \quad (k = 2, 3, \dots, K), \end{cases} \tag{15}$$

where $C_j^n = \dfrac{j!}{(j-n)!n!}$.

Proof. Loosing no generality, for a simple and analytical explanation, I consider the special case of K independent Poisson inputs $\lambda_1, \lambda_2, \dots, \lambda_k, \dots, \lambda_K$ that are offered to the system defined in Theorem 1. Denote $\alpha_k = \sum_{i=k}^{K} \lambda_i$ the birth rate of the system in state $0 \le j \le n_k$, $(1 \le k \le K)$, $1/\mu$ the mean service time of a customer and $\beta_k = \alpha_k / \mu$, $\rho_k = \lambda_k / \mu$. Then, according to the expression of a birth-death process, the steady probability of the system should be

$$\pi(j) = \begin{cases} \dfrac{\beta_1^j}{j!} & 0 \le j \le n_1, \\ \dfrac{\beta_k^{j-n_{k-1}} \prod_{i=1}^{k-1} \beta_i^{n_i - n_{i-1}}}{j!}, & n_{k-1} < j \le n_k, \quad (2 \le k \le K), \end{cases} \tag{16}$$

where assume $n_0 = 0$ and the marginal probability of each traffic stream should be

$$p_k(j) = \begin{cases} \dfrac{\rho_1^j}{j!}, & 0 \le j \le n_k, \\ 0, & \text{otherwise}. \end{cases} \tag{17}$$

For $0 \le j \le n_k$, $(1 \le k \le K)$, as

$$\frac{\beta_k^j}{j!} = \frac{(\rho_k + \rho_{k+1} + \cdots + \rho_K)^j}{j!} = \sum_{m_k + m_{k+1} + \cdots + m_K = j} \prod_{i=k}^{K} \frac{\rho_i^{m_i}}{m_i!} = p_k \otimes \cdots \otimes p_K = q_{(k)}(j),$$

then the convolution law introduced in subsection 3.1 is valid for the traffic streams $\rho_k, \rho_{k+1}, \dots, \rho_K$. Eq. (14) is proved.

According to the expression of Eq. (16),

$$\pi(j)=\frac{\beta_1^j}{j!}=q_{(1)}(j), \qquad 0\le j\le n_1,$$

$$\pi(j)=\frac{\beta_2^{j-n_1}\beta_1^{n_1}}{j!}=\frac{n_1!(j-n_1)!}{j!}q_{(2)}(j-n_1)\pi(n_1)=\frac{q_{(2)}(j-n_1)\pi(n_1)}{C_j^{n_1}}, \quad n_1\le j\le n_2.$$

Therefore the theorem is valid for $0\le j\le n_2$.

The argument for the general cases of $0\le j\le n_k$, $(3\le k\le K)$ is similar to the case of k=1,2. Assume that the theorem is valid for $k=l$, $(l\ge 3)$, then imitate the above argument to show that the theorem is valid for $k=l+1$. This completes the proof. □

Clearly, for the known $p_k(j)$, $(0\le j\le n_k)$, by successively applying Eq. (14), $q_{(k)}$, $(1\le k\le K-1)$ can be determined in $O(n_k^2)$ times and the effort to get all q is then $O(\sum_{k=1}^{K-1}n_k^2)$. Similarly, for the known $q_{(k)}$, $(1\le k\le K)$, $\pi(j)$ can be determined in $O(N-n_1)$ times through Eq. (15). Thus, to determine π, the algorithm is very efficient.

3.4 State Probability of the Mutual Overflow System

Now we come back to the original mutual overflow system introduced in subsection 2.1. The following, we denote again the system state in another way as $Y=(y_1,y_2,y_3,y_4,y_5,y_6)$, where $0\le y_m\le N_m$ (m=1,2,3) represents the number of busy servers in three real facilities and $0\le y_{m+3}\le M_m$ (m=1,2,3) represents what in three virtual facilities, respectively. It is instructive to consider the state x_m (m=1,2,3), (x_m was defined in subsection 2.1) to be the state of link m and y_l to be the state of routing l (l=1,2,...,6) respectively as they are often used in modeling of dynamic routing networks [5]. Now, define an indicator $\theta_{m,l}$ as in Tab.1, State Y should be subject to the following constrain Ψ as

Tab.1 Value of indicator $\theta_{m,l}$.

m \ l	1	2	3	4	5	6
1	1	0	0	0	1	1
2	0	1	0	1	0	1
3	0	0	1	1	1	0

$$Y\in\Psi=\{\,y_l \mid \sum_{l=1}^{6}y_l\theta_{m,l}\le N_m,\quad y_l\ge 0,\quad y_{m+3}\le M_m,\qquad \forall m=1,2,3,\quad l=1,2,...,6\}.$$

Let $X=[x_1,x_2,x_3]^T$, $Y=[y_1,y_2,y_3,y_4,y_5,y_6]^T$ and $A=[\theta_{m,l}]_{3\times 6}$ Therefore, the relationship of state X and Y is then connected by $\theta_{m,l}$ with the expression $X=AY$. For a given state X, denote $\Omega_X\subset\Psi$ representing a subset of state Y which could derives state X, therefore, we give the following theorem.

[Theorem 2] *Let the unnormalized joint probability of a facility and its virtual facility be* $q_m(y_m,y_{m+3})$ *which has been gotten from* ***Algorithm 1****, then for a given state X, the joint probability* $\pi(X)=\pi(x_1,x_2,x_3)$ *of the overall system can be obtained by following formula.*

$$\pi(X)=\sum_{Y\in\Omega_X}q_1(y_1,y_4)q_2(y_2,y_5)q_3(y_3,y_6)D(X,Y)\ , \tag{18}$$

where

$$D(X,Y)=d_2(x_2,y_4)d_3(x_3,y_4)d_1(x_1,y_5)d_3(x_3,y_5)d_1(x_1,y_6)d_2(x_2,y_6), \tag{19}$$

$$d_m(x,y)=\begin{cases}\dfrac{C_{r_m}^y}{C_x^y}, & 0\le y\le r_m<x,\\ 1, & \text{otherwise.}\end{cases} \tag{20}$$

Proof. As the numbering of the traffic streams is indifferent, then we can truncate the state space, therefor both of commutative and associative laws are valid for the productive expression. By combining with the details explained in Fig. 1 and flowing the proof of Theorem 1 in subsection 3.3, the necessary parameter $d_m(x,y)$ for the joint probabilities of the system state is then obtained. □

According to the definition of equation (18), $\pi(X)$ is in general not normalized. Let the normalization factor is G, then the expected result should be $\pi(X)G$.

4. Performance Evaluation Measures

A particular interest for the loss systems is the determination of the blocking probability (the equilibrium customer (call) rejection probability), the utilization of facilities and carried traffic etc.. Based on the result of steady state probabilities in exact way, it is possible in principle to determine these useful performance parameters exactly. For a preparation, we first define the marginal probability for a given facility m or a given pair of facilities i and j as

$$p_m(x_m)=\sum_{x_i=0}^{N_i}\sum_{x_j=0}^{N_j}\pi(X), \qquad 0\le x_m\le N_m, \quad m,i,j=1,2,3, \quad m\ne i\ne j; \tag{21}$$

$$p_{i,j}(x_i,x_j)=\sum_{\substack{x_m=0\\(m\ne i\ne j)}}^{N_m}\pi(X), \qquad 0\le x_i\le N_i, \quad 0\le x_j\le N_j, \quad m,i,j=1,2,3. \tag{22}$$

We de-convolute $\pi(X)$ into $\pi_{X|m}$ and $q_m(y_m,y_{m+3})$ so that $\pi(X)=\pi_{X|m}\otimes q_m$ for any given m (=1,2,3). By caring the convolution with

$$\pi(X)=\sum_{y_m=0}^{x_m}\sum_{y_{m+3}=0}^{u_m(X)}\pi_{X|m}(X-Y_m)q_m(y_m,y_{m+3})=\sum_{y_m=0}^{x_m}\sum_{y_{m+3}=0}^{u_m(X)}b_{X|m}(y_m,y_{m+3}), \quad m=1,2,3, \tag{23}$$

where $u_m(X)=\min\{\min_{i\ne j\ne m}\{x_i,x_j\},M_m\}$,

$(X-Y_m)=(x_m-y_m,x_i-y_{m+3},x_j-y_{m+3})$,

I then obtain the following performance measures to evaluate the system.

(1) Blocking probability for first-offered traffic m:

$$B_m=G\cdot\left[\sum_{x_i=r_i+1}^{N_i}p_{m,i}(N_m,x_i)+\sum_{x_j=r_j+1}^{N_j}p_{m,j}(N_m,x_j)-\sum_{x_i=r_i+1}^{N_i}\sum_{x_j=r_j+1}^{N_j}\pi(N_m,x_i,x_j)\right],$$

$$i\ne j\ne m, \quad i,j,m=1,2,3. \tag{24}$$

As the arrival process is Poisson, then PASTA is valid, therefore above blocking probability equals to both time congestion and customer (call) congestion. The latter represents the event that when a customer from traffic m arrivals, all the available servers are occupied.

(2) Let e_m be the carried traffic m by the system,

$$e_m=G\cdot\sum_X\sum_{y_m=0}^{x_m}\sum_{y_{m+3}=0}^{u_m(X)}(y_m+y_{m+3})\cdot b_{X|m}(y_m,y_{m+3}), \qquad m=1,2,3. \tag{25}$$

If the first-offered traffic m is $\rho_m=\lambda_m/\mu$, then its traffic congestion is

$$E_m=1-e_m/\rho_m, \qquad m=1,2,3. \tag{26}$$

(3) Utilization η_0 of the overall system is given by

$$\eta_0=\frac{e_1+e_2+e_3}{N_1+N_2+N_3}. \tag{27}$$

(4) Carried total traffic by facility m is

$$\varepsilon_m=G\cdot\sum_{x_m=0}^{N_m}x_m\cdot p_m(x_m), \qquad (m=1,2,3). \tag{28}$$

(5) Utilization η_m of facility m is

$$\eta_m = N_m^{-1}\left(a_m + \sum_{i(i\neq m)}(e_i - a_i)/2\right) = 2(e_m + \varepsilon_m) - \sum_{i(i\neq m)}(\varepsilon_i - e_i)/4N_m, \qquad (29)$$

where $a_m = e_m - \sum_{i(i\neq m)} \varepsilon_i - e_i/2$ is the carried traffic m by facility m with known e_i and ε_i from equations (25) and (28). It can be obtained by solving following equation set

$$a_m + \sum_{i(i\neq m)}(e_i - a_i) = \varepsilon_m, \qquad (m = 1,2,3). \qquad (30)$$

In equation (29), the second item inner the bracket is carried overflow traffic by facility m. Because overflow is simultaneously carried by two servers belonging to different facilities, then the contribution for caring this traffic by facility m is reduced by half.

5. Numerical Results

In this section, we consider an example of a symmetric system by setting $\lambda_1 = \lambda_2 = \lambda_3 = 10$, $N_1 = N_2 = N_3 = 15$ and μ=1 to show the effectiveness of the presented analysis method. The discussion will be based on the variations of server reservation threshold in facility 1 with $0 \le r_1 = r \le N_1$, and that in facilities 2 and 3 with simultaneous adjustment of $0 \le r_2 = r_3 = R \le N_2 (= N_3)$. Thus we are able to describe some of the major performance measures in three-dimensional figures as follows. In Fig. 5, 6 and 7 the horizontal axis from left to right represents the changes of r, and the horizontal axis from far to near represents the simultaneous changes R, and the vertical axis represents focused performance measures.

For the parameter's choice $r_1 = r_2 = r_3 = 0$, the solution is the product form of the marginal Erlang's formulas for the three systems as they now behave completely independent. On the other hand, partial or complete sharing is obtained for the choice of $0 < r_m \le N_m$ $(m = 1,2,3)$.

According to Fig. 5(a), a welcome result is that with the increases of R, the blocking of first-offered traffic 1 becomes low and it seems to be not so sensitive with the changes of r of its self. The first conclusion of the above result shows us that the powerful effect of load balance mechanism. While the second phenomenon is partly because that the other demands from overflow for the server of facility 1 are dominated by the minimum one between r and R. Then even r increased, the exceeded parts beyond R will take no influence to overflow demand. These second phenomena could be given a further explanation according to Fig. (b)s, where we can see clearly the influence to the mentioned parameters caused by two partner facilities that are now joined together for the simultaneous occupation of overflow.

An important result is derived from Fig. 6 that with the increases of r and R, the utilization of each facility becomes lower and lower. This is coincident with our common sense that system utilization will deteriorate as the degree of resource sharing increases under overload situation. The reason for this phenomenon could be explained as since as double as many resources have to be put into caring overflow traffic, the simultaneous resource occupation causes the inferiority of system utilization.

It is also noticed that small reservation parameters have a correspondingly small effect on total system performance and can therefore be used to carry overflow traffic without too much worry.

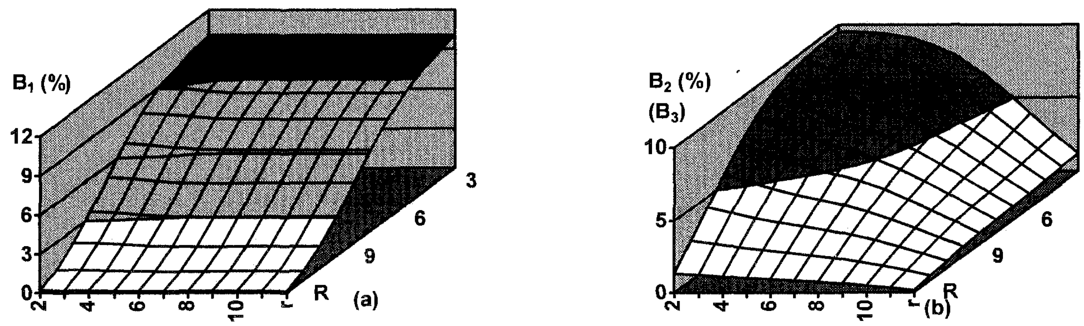


Fig.5 Blocking of traffic 1 and 2 or 3 vs. r and R.

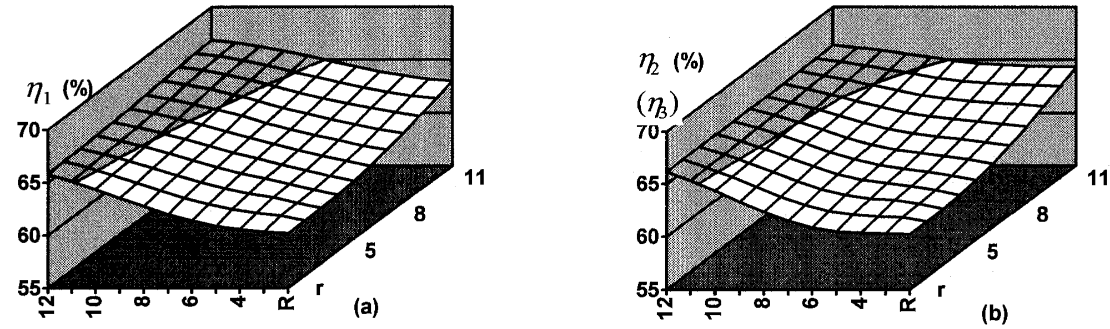


Fig.6 Utilization of Facilities 1 and 2 or 3 vs. r and R.

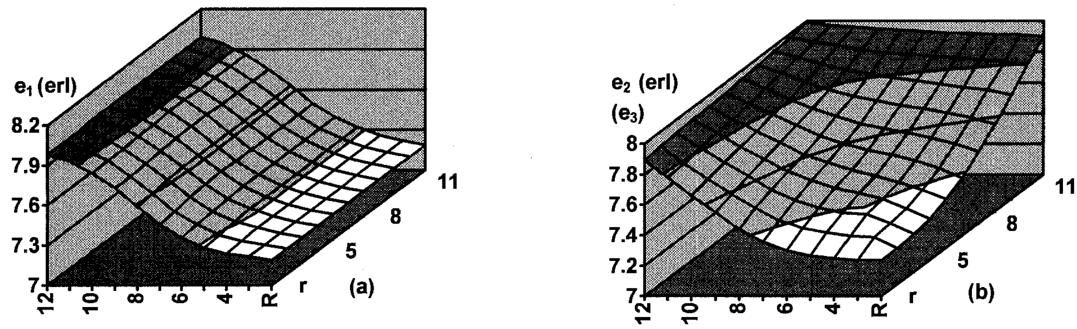


Fig.7 Carried traffic 1 and 2 or 3.

6. Summary and Conclusion

In this paper, by extending known results, we developed a realistic algorithm and a set of performance evaluation formulas to exactly analyze the load balance performance in a mutual overflow system with simultaneous occupation of resources. The relative numerical studies shown that they could help us to know exactly more inherent characteristics about load balance mechanism in the circumstances with simultaneous resource possession. The common characteristic of above numerical results could immediately lead us to the conclusion that the real benefit from introducing dynamic routing technology to a telecommunications network is fair allocation of network resources for all the traffic streams in unbalanced load environment. This benefit is obtained by scarifying network utilization. It is then declared that more sharing by overflow traffic will result low utilization of resources and may lead to the fatal dead lock in a network due to the overload spread. Therefore the network should be carefully under control to protect against the occurrence of more new overflow caused by the sharing of network resources by other overflow traffic.

It is clear that the idea for getting the presented models and algorithms in this paper may also be put into the studies of a queuing system with more than three facilities. In that case, for route overflow traffic, man has to make decisions for selecting secondary facilities. Then the associate discussion would be combined with the aspect of optimization which is the problem of optimal traffic allocation control or routing decision-making policies.

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